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Extra Spin-Rotation Coupling Effect in a Radiating Kerr Space-time

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Source-less wave equations are derived for massless scalar, neutrino and electromagnetic perturbations of a radiating Kerr space-time, and the Hawking radiation of massless particles with spin $s = 0, 1/2$ and 1 in this geometry is investigated by using a method of the generalized tortoise coordinate transformation. An extra interaction between the spin of particles and the rotation of the hole displays in the thermal spectra of Hawking radiation of massless particles with spin $s = 1/2, 1$ in the evaporating Kerr space-time. The character of such effect is its obvious dependence on different helicity states of particles with higher spin.

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It is generally accepted in black-hole physics that the total mass-energy of a rotating charged black hole can be separated into three parts [1]: the rotational energy, the Coulomb energy and the irreducible mass, which was reinterpreted later by Smarr [2] as the surface energy. Correspondingly, the energy of a charged particle in a charged axisymmetric black hole is composed of the electromagnetic interaction part and the energy due to the coupling of the orbital angular momentum of a particle with the rotation of the black hole. Besides these two ingredients, one can expect naively that there are other interaction energies such as those arising from the gravitational coupling of rotation or acceleration of a black hole with the intrinsic spin of a spinning particle [3]. Unless one puts by hand the spin-rotation coupling term into the Hamiltonian of the system or takes into account the higher order spin relativistic effects in a non-relativistic approximation [3], this is not the case because the Hawking radiation radiation [4] spectra show that there is no such thing in a stationary rotating black hole background. The well-known thermal radiation spectra [5,6] of all particle species in a stationary space-time are given by

$$\langle \mathcal{N}_\omega \rangle \sim \{\exp[(\omega - m\Omega_h - q\Phi_h)/T_h] \pm 1\}^{-1} , \qquad (1)$$

where three constants Ω_h, Φ_h, T_h are, respectively, the angular velocity, the electromagnetic potential and the effective temperature of the event horizon of the hole, while m is the azimuthal quantum number of the particle, q its charge. The spectrum formula of Eq. (1) can be easily derived from the Teukolsky master equation [7] in the Kerr space-time [8] by means of the method suggested by Damour-Ruffini and Sannan [5].

Situation may be changed if one turns to consider the Hawking radiation of particles with higher spin in a dynamic black hole. In recent papers [9], the evaporation of Dirac particles in a variable-mass Kerr black hole [10,11] has been discussed by use of a method of generalized tortise coordinate transformation (GTCT). A new interaction effect due to the coupling of the intrinsic spin of Dirac particles with the angular momentum of the radiating Kerr black hole was observed in the thermal radiation spectrum of particles with spin-1/2. The character of this spin-rotation coupling effect is its obvious dependence on different helicity states of the spinning particles. This effect disappears [12] when the space-time degenerates to a spherically symmetric black hole of Vaidya-type. It should be noted that this effect displayed in the Fermi-Dirac spectrum is absent in the Bose-Einstein distribution of Klein-Gordon particles.

The aim of this letter is twofold. First, we deduce the perturbed wave equations for scalar, neutrino and electromagnetic fields in an evaporating Kerr black hole [10,11]. It should be noted that the Teukolsky's context [7] on the perturbations of the Kerr black hole [8] can not apply here to the gravitational perturbation of this space-time because it is of Petrov type-II. We point out that the black hole radiation of the Vaidya metric has already been studied [13] within the Teukolsky's celebrated framework [7]. Second, we deal with the thermal radiation of particles with spin-$0, 1/2$ and 1 in the non-stationary Kerr black hole. The method used here is just the same as that we have developed in [9] to investigate the Hawking effect of Dirac particles in this space-time. We find that in the Hawking radiation spectra of spinning particles, there appears an extra interaction energy due to the coupling of the intrinsic spin of particles with the rotation of the black hole. This effect vanishes when the space-time becomes a stationary Kerr black hole or a Vaidya-type spherically symmetric black hole.

The metric describing a radiating Kerr black hole can be written in the advanced Eddington-Finkelstein coordinates system as [10,11]

$$\begin{aligned} ds^2 = &\frac{\Delta - a^2\sin^2\theta}{\Sigma}dv^2 - 2dvdr + 2a\sin^2\theta drd\varphi \\ &- \Sigma d\theta^2 + 2\frac{r^2 + a^2 - \Delta}{\Sigma}a\sin^2\theta dvd\varphi \\ &- \frac{\left(r^2 + a^2\right)^2 - \Delta a^2\sin^2\theta}{\Sigma}\sin^2\theta d\varphi^2 , \end{aligned} \qquad (2)$$

where $\Delta = r^2 - 2M(v)r + a^2$, $\Sigma = r^2 + a^2\cos^2\theta = \rho^*\rho$, $\rho = r + ia\cos\theta$, and v is the standard advanced time. The line element is a natural non-stationary generalization of

the stationary Kerr solution. The mass M depends on the time v, but the specific angular momentum a is a constant.

The space-time geometry of an evaporating Kerr black hole is characterized by three kinds of surfaces of particular interest: the apparent horizons $r_{AH}^{\pm} = M \pm \sqrt{M^2 - a^2}$, the timelike limit surfaces $r_{TLS}^{\pm} = M \pm \sqrt{M^2 - a^2\cos\theta}$, and the event horizons $r_{EH}^{\pm} = r_H$. The event horizon is necessary a null-surface $r = r(v, \theta)$ that satisfies the null-surface conditions $g^{\mu\nu}\partial_\mu F \partial_\nu F = 0$ and $F(v, r, \theta) = 0$.

The traditional method to determine the location and the temperature of the event horizon of a dynamic black hole is to calculate vacuum expectation value of the renormalized energy momentum tensor [14]. But this method is very complicated and meets great difficulties here. A more effective method is called the generalized tortoise coordinate transformation (GTCT) which can give simultaneously the exact values both of the location and of the temperature of the event horizon of a non-stationary black hole. Basically, this method is to reduce Klein-Gordon or Dirac equation in a known black hole background to a standard wave equation near the event horizon by generalizing the common tortoise-type coordinate $r_* = r + \frac{1}{2\kappa}\ln(r - r_H)$ in a static or stationary space-time [5] (where κ is the surface gravity of the studied event horizon) to a similar form in a non-static or non-stationary space-time and by allowing the location of the event horizon r_H to be a function of the advanced time $v = t + r_*$ and/or the angles θ, φ.

As the space-time under consideration is symmetric about φ-axis, one can introduce the following generalized tortoise coordinate transformation (GTCT) [9]

$$\begin{aligned} r_* &= r + \frac{1}{2\kappa}\ln\left[r - r_H(v, \theta)\right], \\ v_* &= v - v_0, \quad \theta_* = \theta - \theta_0, \end{aligned} \tag{3}$$

where $r_H = r(v, \theta)$ is the location of event horizon, and κ is an adjustable parameter. All parameters κ, v_0 and θ_0 characterize the initial state of the hole and are constant under the tortoise transformation.

Substituting the GTCT of Eq. (3) into the null-surface equation $g^{\mu\nu}\partial_\mu F \partial_\nu F = 0$ and then taking the $r \to r_H(v_0, \theta_0)$, $v \to v_0$ and $\theta \to \theta_0$ limits, we can arrive at

$$\left[\Delta_H - 2\big(r_H^2 + a^2\big)\dot{r}_H + a^2\sin^2\theta_0\dot{r}_H^2 + r_H'^2\right]\Big(\frac{\partial F}{\partial r_*}\Big)^2 = 0\,, \tag{4}$$

in which the vanishing of the coefficient in the square bracket can give the following equation to determine the location of the event horizon of an evaporating Kerr black hole

$$\Delta_H - 2\big(r_H^2 + a^2\big)\dot{r}_H + a^2\sin^2\theta_0\dot{r}_H^2 + r_H'^2 = 0\,, \tag{5}$$

where we denote $\Delta_H = r_H^2 - 2M(v_0)r_H + a^2$. The quantities $\dot{r}_H = \partial_v r_H$ and $r_H' = \partial_\theta r_H$ depict the change of the event horizon in the advanced time and with the angle, which reflect the presence of quantum ergosphere near the event horizon. Eq. (5) means that the location of the event horizon is shown as

$$\begin{aligned} r_H = \big\{M \pm \big[M^2 - \big(a^2\sin^2\theta_0\dot{r}_H^2 + r_H'^2\big)\big(1 - 2\dot{r}_H\big) \\ - a^2\big(1 - 2\dot{r}_H\big)^2\big]^{1/2}\big\}/(1 - 2\dot{r}_H)\,. \end{aligned} \tag{6}$$

To derive the scalar, neutrino and electromagnetic perturbations in the evaporating Kerr black hole, we establish a complex null-tetrad which has $[v, r, \theta, \varphi]$ components: $l^\mu = -\delta_1^\mu$, $n^\mu = \big[\big(r^2 + a^2\big)\delta_0^\mu + 2^{-1}\Delta\delta_1^\mu + a\delta_3^\mu\big]/\Sigma$, $m^\mu = \big(ia\sin\theta\delta_0^\mu + \delta_2^\mu + i\delta_3^\mu/\sin\theta\big)/\big(\sqrt{2}\rho\big)$. Within the Newman-Penrose (NP) formalism [15], we substitute, respectively, $\chi_1 = \sqrt{2\Sigma}\eta_1\,, \chi_0 = \sqrt{\Sigma}\eta_0/\rho^*$ for the Weyl spinors η_0, η_1 and $\Phi_0 = \rho\phi_0/\big(\sqrt{2}\rho^*\big)\,, \Phi_1 = \rho\phi_1\,, \Phi_2 = \sqrt{2}\Sigma\phi_2$ for the Maxwell complex scalars ϕ_0, ϕ_1, ϕ_2 into the NP forms of both Weyl equation and Maxwell equation [7], and get the following first-order equations

$$\partial_r\chi_1 + \mathcal{L}_{1/2}\chi_0 = 0\,, \quad \Delta\mathcal{D}_{1/2}\chi_0 - \mathcal{L}^\dagger{}_{1/2}\chi_1 = 0\,, \tag{7}$$

for the Weyl neutrinos, and

$$\begin{aligned} &\big(\partial_r + 1/\rho\big)\Phi_1 + \big(\mathcal{L}_1 + ia\sin\theta/\rho\big)\Phi_0 = 0\,, \\ &\Delta\big(\mathcal{D}_1 - 1/\rho\big)\Phi_0 - \big(\mathcal{L}^\dagger{}_0 - ia\sin\theta/\rho\big)\Phi_1 = 0\,, \\ &\big(\partial_r - 1/\rho\big)\Phi_2 + \big(\mathcal{L}_0 - ia\sin\theta/\rho\big)\Phi_1 = 0\,, \\ &\Delta\big(\mathcal{D}_0 + 1/\rho\big)\Phi_1 - \big(\mathcal{L}^\dagger{}_1 + ia\sin\theta/\rho\big)\Phi_2 \\ &\quad = 2i\dot{M}ra\sin\theta\Phi_0\,, \end{aligned} \tag{8}$$

for the photons. Here we have defined operators $\mathcal{D}_n = \partial_r + 2\Delta^{-1}\left[n(r - M) + a\partial_\varphi + \big(r^2 + a^2\big)\partial_v\right]$, $\mathcal{L}_n = \partial_\theta + n\cot\theta - \frac{i}{\sin\theta}\partial_\varphi - ia\sin\theta\partial_v$, and the complex conjugate $\mathcal{L}^\dagger{}_n$ for operator $\mathcal{L}_n$.

Eqs. (7,8) can not be decoupled except in the stationary Kerr black hole case ($M = const$) or in a Vaidya-type space-time. However, to deal with the problem of Hawking radiation, one may concern about their asymptotic behaviors near the horizon only. Making use of the transformation of Eq. (3) and then taking the $r \to r_H(v_0, \theta_0)$, $v \to v_0$ and $\theta \to \theta_0$ limits, one can reduce Eqs. (7,8) to the following forms

$$\begin{aligned} &\frac{\partial\Psi_{p+1}}{\partial r_*} - \Big(r_H' - ia\sin\theta_0\dot{r}_H\Big)\frac{\partial\Psi_p}{\partial r_*} = 0\,, \\ &\Big(r_H' + ia\sin\theta_0\dot{r}_H\Big)\frac{\partial\Psi_{p+1}}{\partial r_*} \\ &\quad + \Big[\Delta_H - 2\big(r_H^2 + a^2\big)\dot{r}_H\Big]\frac{\partial\Psi_p}{\partial r_*} = 0\,, \end{aligned} \tag{9}$$

where Ψ_p stands for the Weyl spinors χ_0, χ_1 when $p = 0, 1 (s = 1/2)$ and represents the Maxwell scalars Φ_0, Φ_1, Φ_2 when $p = 0, 1, 2 (s = 1)$. The existence condition of nontrial solutions for Ψ_p is that the determinant of Eq. (9) vanishes which results in the event horizon equation (5) given above.

With the first-order equations (7, 8) in hand, we are now in a position to derive their corresponding second-order equations

$$\begin{aligned}&\big(\partial_r\Delta\mathcal{D}_{1/2}+\mathcal{L}^{\dagger}{}_{1/2}\mathcal{L}_{1/2}\big)\chi_0=0\,,\\&\big(\Delta\mathcal{D}_{1/2}\partial_r+\mathcal{L}_{1/2}\mathcal{L}^{\dagger}{}_{1/2}\big)\chi_1\\&\quad=ia\sin\theta\big(2\dot{M}r\partial_r+\dot{M}\big)\chi_0\,,\end{aligned}\qquad(10)$$

and

$$\begin{aligned}&\big(\partial_r\Delta\mathcal{D}_1+\mathcal{L}^{\dagger}{}_0\mathcal{L}_1+2\rho^*\partial_v\big)\Phi_0=0\,,\\&\big(\partial_r\Delta\mathcal{D}_0+\mathcal{L}^{\dagger}{}_1\mathcal{L}_0-2\rho^*\partial_v+2M\rho^*/\rho^2\big)\Phi_1\\&\quad=2i\dot{M}ra\sin\theta\big(\partial_r-1/\rho\big)\Phi_0\,,\\&\big(\Delta\mathcal{D}_0\partial_r+\mathcal{L}_0\mathcal{L}^{\dagger}{}_1-2\rho^*\partial_v\big)\Phi_2\\&\quad=-2\ddot{M}ra^2\sin^2\theta\Phi_0-4i\dot{M}ra\sin\theta\mathcal{L}_1\Phi_0\,.\end{aligned}\qquad(11)$$

The source-less wave equation $\Box\Phi=0$ for a massless scalar field can be written as

$$\big(\Delta\mathcal{D}_1\partial_r+\mathcal{L}_1\mathcal{L}^{\dagger}{}_0+2\rho^*\partial_v\big)\Phi=0\,.\qquad(12)$$

These equations can be thought of as the generalized Teukolsky master equations, they encompass the well-known results when the mass of the black hole is a constant [7] or the black hole is non-rotating [13]. Because the radiating Kerr metric is of Petrov type-II, the gravitational perturbations of this space-time is more involved. It can be done for the ψ_0,ψ_1 components but not easily for the other components of the Weyl tensors.

Having derived the master equations controlling the scalar, neutrino and electromagnetic perturbations of the non-stationary Kerr metric, we are now ready to study the quantum thermal property of this space-time by investigating the Hawking radiation of massless particles with spin-$0,1/2$ and 1. Given the GTCT in Eq. (3), the limiting form of Eqs. (10-12), when r approaches $r_H(v_0,\theta_0)$, v goes to v_0 and θ goes to θ_0, can be recast into the standard wave equation near the event horizon in an united form

$$\begin{aligned}&\frac{\partial^2\Psi_p}{\partial r_*^2}+2\frac{\partial^2\Psi_p}{\partial r_*\partial v_*}+2\Omega\frac{\partial^2\Psi_p}{\partial r_*\partial\varphi}\\&\quad+2C_3\frac{\partial^2\Psi_p}{\partial r_*\partial\theta_*}+2\big(C_2+iC_1\big)\frac{\partial\Psi_p}{\partial r_*}=0\,,\end{aligned}\qquad(13)$$

where Ω, C_3, C_2 and C_1 are all real constants,

$$\begin{aligned}&\Omega=a(1-\dot{r}_H)/\big[r_H^2+a^2-\dot{r}_Ha^2\sin^2\theta_0\big]\,,\\&C_3=-r'_H/\big[r_H^2+a^2-\dot{r}_Ha^2\sin^2\theta_0\big]\,,\\&C_2+iC_1=\frac{-1}{2\big(r_H^2+a^2-\dot{r}_Ha^2\sin^2\theta_0\big)}\Big[2p\big(r_H-M\\&+ia\cos\theta_0\dot{r}_H\big)-2(2p+1)r_H\dot{r}_H+\ddot{r}_Ha^2\sin^2\theta_0+r''_H\\&+\cot\theta_0r'_H+2(s+p)\dot{M}r_H\frac{a^2\sin^2\theta_0\dot{r}_H-ia\sin\theta_0r'_H}{\Delta_H-2\dot{r}_H\big(r_H^2+a^2\big)}\Big]\,.\end{aligned}$$

The wave equation (13) includes the scalar field Φ as a special case when $p=s=0$. Also it holds true for a propagating external gravitational field with suitable replacements of all field components.

In deriving Eq. (13), we have dealt with an infinite form of 0/0-type in order to obtain a finite value $2(r_H-M)-4r_H\dot{r}_H$ and adjusted the parameter κ such that it satisfies

$$\begin{aligned}&\frac{r_H\big(1-2\dot{r}_H\big)-M}{\kappa}+2\Delta_H-2\dot{r}_H\big(r_H^2+a^2\big)\\&\quad=r_H^2+a^2-\dot{r}_Ha^2\sin^2\theta_0\,,\end{aligned}\qquad(14)$$

which means the surface gravity is

$$\kappa=\frac{r_H\big(1-2\dot{r}_H\big)-M}{\big(r_H^2+a^2-\dot{r}_Ha^2\sin^2\theta_0\big)\big(1-2\dot{r}_H\big)+2r'^2_H}\,.\qquad(15)$$

The location (6) and the temperature $\kappa/(2\pi)$ of event horizon of a non-stationary Kerr black hole are shown to be dependent not only on the advanced time v but also on the angle θ. Both of them can be calculated by iteration, but we don't pursue this goal here. A crucial step of our treatment is to use the relations between the first-order derivatives (9) to eliminate the crossing-term of the first-order derivatives in the second-order equations near the event horizon. It is also important to adjust the parameter κ so as to recast each second-order equation into a standard wave equation near the event horizon.

Now that all real coefficients in Eq. (13) can be regarded as finite constants, one can separate variables as $\Psi_p=R(r_*)\exp[\lambda\theta_*+i(m\varphi-\omega v_*)]$ and have a solution to the radial part as $R=R_1\exp[2i(\omega-m\Omega-C_1+iC_0)r_*]+R_0$, in which λ is a real separation constant, $C_0=\lambda C_3+C_2$. The ingoing wave $\Psi_p^{\rm in}\sim\exp[\lambda\theta_*+i(m\varphi-\omega v_*)]$ is regular at the event horizon $r=r_H$, whereas the outgoing wave

$$\Psi_p^{\rm out}(r>r_H)=\Psi_p^{\rm in}\exp[2i(\omega-m\Omega-C_1+iC_0)r_*]\,,\qquad(16)$$

is irregular, it can be analytically continued from the outside of the hole into the inside of the hole through the lower complex r-plane to

$$\widetilde{\Psi_p^{\rm out}}(r<r_H)=\Psi_p^{\rm out}\exp[\pi(\omega-m\Omega-C_1+iC_0)/\kappa]\,.\qquad(17)$$

According to the method of Damour-Ruffini-Sannan's [5], the relative scattering probability at the event horizon and the thermal radiation spectrum of particles from the black hole are, respectively,

$$\begin{aligned}&\Big|\frac{\Psi_p^{\rm out}}{\widetilde{\Psi_p^{\rm out}}}\Big|^2=\exp[-2\pi(\omega-m\Omega-C_1)/\kappa]\,,\\&\langle\mathcal{N}(\omega)\rangle=\frac{\Gamma(\omega)}{\exp[2\pi(\omega-m\Omega-C_1)/\kappa]-(-1)^{4s^2}}\,,\end{aligned}\qquad(18)$$

where $\Gamma(\omega)$ is the transmission coefficient in certain modes with which a particle can escape from the event horizon to infinity, m the azimuthal quantum number. Ω can be interpreted as the angular velocity of the event horizon of the evaporating Kerr black hole, while the explicit expression of the "spin-dependent" term C_1 reads

$$C_1 = \frac{1}{r_H^2 + a^2 - \dot{r}_H a^2 \sin^2\theta_0}\Big[-pa\cos\theta_0 \dot{r}_H + (s+p)\frac{\dot{M} r_H a \sin\theta_0 r_H'}{\Delta_H - 2\dot{r}_H(r_H^2+a^2)}\Big]. \quad (19)$$

The thermal radiation spectrum (18) of particles with spin-s is composed of two parts: $\omega_p = m\Omega + C_1$, one is the rotational energy $m\Omega$ arising from the coupling of the orbital angular momentum of particles with the rotation of the black hole; another is C_1 due to the coupling of the intrinsic spin of particles and the angular momentum of the hole, which vanishes in the case of a stationary Kerr black hole ($M = const$, $\dot{r}_H = r_H' = 0$) or a Vaidya-type black hole ($a = 0$, $r_H' = 0$, $\dot{r}_H \neq 0$). The distribution (18) recovers the known results [12,13] when the black hole is non-rotating, and approximates the formula (1) when $\dot{r}_H \simeq 0$ (namely, the luminosity $L = -dM/dv = -\dot{M} \simeq 0$). To see its meaning more transparently, let us, of astrophysical interest, neglect the angular deformation r_H' of the hole, then we can approximate ω_p as

$$\omega_p \simeq \big(m - p\frac{\dot{r}_H}{1-\dot{r}_H}\cos\theta_0\big)\Omega\,, \quad p = 0, 1, \cdots, 2s \quad (20)$$

in the case of small evaporation and slow rotation. The factor $\dot{r}_H/(1-\dot{r}_H)$ describes the evolution of the hole in the time, while the factor $\cos\theta_0$ comes from the scalar product of the spin-rotation coupling. The term C_1 is obviously related to the helicity of particles in different spin states, it characterizes a new effect arising from the interaction between the spin of particles and the rotation of a evaporating black hole. The feature of this new effect is that it is dependent on different helicity states of spinning particles.

In summary, this study not only encompasses the thermal spectrum of spinning particles in the Vaidya metric [13], but also provides a partial confirmation of York's conjecture [14] that quantum radiance of a charged and rotating hole might be originated from the quantum ergosphere effect. Here we suggest that the radiative mechanism of an evaporating Kerr black hole can be changed by the quantum rotating ergosphere which can be viewed as a mixture of the classical rotating ergosphere and York's quantum ergosphere. We argue that dynamic black holes must have some new properties very different from that of stationary ones, and the spin-rotation coupling effect presented in this study may be a good example. We suggest a new experiment in which this effect can be observed from the spectrography of certain astrophysical objects in the future.

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